Duality and anyonic excitations

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Abstract

We consider anyonic excitations classified into equivalence classes labeled by Hausdorff dimension, h and introduce the concept of duality between such classes, defined by $\tilde{h} = 3 - h$. In this way, we confirm that the filling factors for which the Fractional Quantum Hall Effect (FQHE) were observed just appears into these classes and the duality in case is between quasihole and quasiparticle excitations for these FQHE systems. Exchanges of dual pairs $(\nu, \tilde{\nu})$, suggests conformal invariance.

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We have classified the fractional spin particles or anyonic excitations into equivalence classes labeled by Hausdorff dimension, h and conjectured in this way a new hierarchy scheme for the filling factors, ν. Such parameters characterize the Fractional Quantum Hall Effect (FQHE) and second our approach we can predicting for which values of ν FQHE can be observed [1]. In [2] we have obtained an anyonic distribution function for each class h, which reduces to bosonic and fermionic distributions, when $h = 2$ and $h = 1$, respectively.

Now, we introduce the concept of duality between equivalence classes, defined by

$$\tilde{h} = 3 - h, \tag{1}$$

such that, for $h = 1$, we have $\tilde{h} = 2$ and for $h = 2$, we have $\tilde{h} = 1$. This means that for h defined into the interval $1 < h < 2$, with h a rational fraction with odd denominator, is related to the filling factor ν as follows:

$$\begin{array}{llll} h_1 = 2 - \nu, & 0 < \nu < 1; & h_2 = \nu, & 1 < \nu < 2; \\ h_3 = 4 - \nu, & 2 < \nu < 3; & h_4 = \nu - 2, & 3 < \nu < 4; \\ h_5 = 6 - \nu, & 4 < \nu < 5; & h_6 = \nu - 4, & 5 < \nu < 6; \\ etc. & & & \end{array} \tag{2}$$

For the set of values of the filling factors ν experimentally observed [3], second our relations (Eq.2 and Eq.1), we get the classes, h and $\tilde{h}$:

$$\begin{array}{ll} \left\{\frac{1}{3}, \frac{5}{3}, \frac{7}{3}, \frac{11}{3}, \cdots\right\}_{h=\frac{5}{3}}, & \left\{\frac{2}{3}, \frac{4}{3}, \frac{8}{3}, \frac{10}{3}, \cdots\right\}_{\tilde{h}=\frac{4}{3}}; \\ \left\{\frac{1}{5}, \frac{9}{5}, \frac{11}{5}, \frac{19}{5}, \cdots\right\}_{h=\frac{9}{5}}, & \left\{\frac{4}{5}, \frac{6}{5}, \frac{14}{5}, \frac{16}{5}, \cdots\right\}_{\tilde{h}=\frac{6}{5}}; \\ \left\{\frac{2}{7}, \frac{12}{7}, \frac{16}{7}, \frac{26}{7}, \cdots\right\}_{h=\frac{12}{7}}, & \left\{\frac{5}{7}, \frac{9}{7}, \frac{19}{7}, \frac{23}{7}, \cdots\right\}_{\tilde{h}=\frac{9}{7}}; \\ \left\{\frac{2}{9}, \frac{16}{9}, \frac{20}{9}, \frac{34}{9}, \cdots\right\}_{h=\frac{16}{9}}, & \left\{\frac{7}{9}, \frac{11}{9}, \frac{25}{9}, \frac{29}{9}, \cdots\right\}_{\tilde{h}=\frac{11}{9}}; \\ \left\{\frac{2}{5}, \frac{8}{5}, \frac{12}{5}, \frac{18}{5}, \cdots\right\}_{h=\frac{8}{5}}, & \left\{\frac{3}{5}, \frac{7}{5}, \frac{13}{5}, \frac{17}{5}, \cdots\right\}_{\tilde{h}=\frac{7}{5}}; \\ \left\{\frac{3}{7}, \frac{11}{7}, \frac{17}{7}, \frac{25}{7}, \cdots\right\}_{h=\frac{11}{7}}, & \left\{\frac{4}{7}, \frac{10}{7}, \frac{18}{7}, \frac{24}{7}, \cdots\right\}_{\tilde{h}=\frac{10}{7}}; \\ \left\{\frac{4}{9}, \frac{14}{9}, \frac{22}{9}, \frac{32}{9}, \cdots\right\}_{h=\frac{14}{9}}, & \left\{\frac{5}{9}, \frac{13}{9}, \frac{23}{9}, \frac{31}{9}, \cdots\right\}_{\tilde{h}=\frac{13}{9}}; \\ \left\{\frac{6}{13}, \frac{20}{13}, \frac{32}{13}, \frac{46}{13}, \cdots\right\}_{h=\frac{20}{13}}, & \left\{\frac{7}{13}, \frac{19}{13}, \frac{33}{13}, \frac{45}{13}, \cdots\right\}_{\tilde{h}=\frac{19}{13}}; \\ \left\{\frac{5}{11}, \frac{17}{11}, \frac{27}{11}, \frac{39}{11}, \cdots\right\}_{h=\frac{17}{11}}, & \left\{\frac{6}{11}, \frac{16}{11}, \frac{28}{11}, \frac{38}{11}, \cdots\right\}_{\tilde{h}=\frac{16}{11}}; \\ \left\{\frac{7}{15}, \frac{23}{15}, \frac{37}{15}, \frac{53}{15}, \cdots\right\}_{h=\frac{23}{15}}, & \left\{\frac{8}{15}, \frac{22}{15}, \frac{38}{15}, \frac{52}{15}, \cdots\right\}_{\tilde{h}=\frac{22}{15}}. \end{array} \tag{3}$$

We note that in each class, the first filling factors are the experimental values observed (also some second and third values) that is, the Hall resistance develops plateaus in these

quantized values, which are related to the fraction of electrons that form collective excitations as quasiholes or quasiparticles in FQHE systems. The relation of duality between equivalence classes labeled by h, therefore, can indicate a way as determine the dual of a specific value of ν (or $\tilde{\nu}$) observed. On the other hand, for excitations above the Laughlin ground state, the exchange of two quasiholes [4] with coordinates z_α and z_β produces the condition on the phase

$$\exp\left\{\imath\pi\nu_1\right\} = \exp\left\{\imath\pi\frac{1}{m}\right\}, \tag{4}$$

with $\nu_1 = \frac{1}{m} + 2p_1$; and for a second generation of quasihole excitations, the effective wave-function carries the factor $(z_\alpha - z_\beta)^{\nu_2}$, with $\nu_2 = \frac{1}{\nu_1} + 2p_2$; $m = 3, 5, 7, \cdots$ and p_1, p_2 are positive integers. In [5] we have noted that these conditions over the filling factor ν confirm our classification of the collective excitations in terms of h. Another interesting point is that in each class, we have more filling factors which those generate by (Eq.4), that is, our classification cover the complete spectrum of states. Now, we can see that the duality between equivalence classes means duality between quasiholes and quasiparticles, that is, between h and $\tilde{h}$. Therefore, as we said elsewhere [1], h tell us about the nature of the anyonic excitations.

On the other hand, we note that the anyonic exchanges of duals get a phase difference, modulo constant,

$$|\nu - \tilde{\nu}| = |\Delta\nu| = h - \tilde{h} = const, \tag{5}$$

suggesting an invariance, conformal symmetry. We have also for the elements of h and $\tilde{h}$, the following relations

$$\begin{aligned} \frac{\nu_{i+1} - \nu_i}{\tilde{\nu}_{j+1} - \tilde{\nu}_j} &= 1; \\ \frac{\nu_{j+1} - \nu_j}{\tilde{\nu}_{i+1} - \tilde{\nu}_i} &= 1, \end{aligned} \tag{6}$$

with $i = 1, 3, 5, etc.$ and $j = 2, 4, 6, etc.$; such that the pairs $(i, j) = (1, 2), (3, 4), etc.$ satisfy the expressions (Eq.6).

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